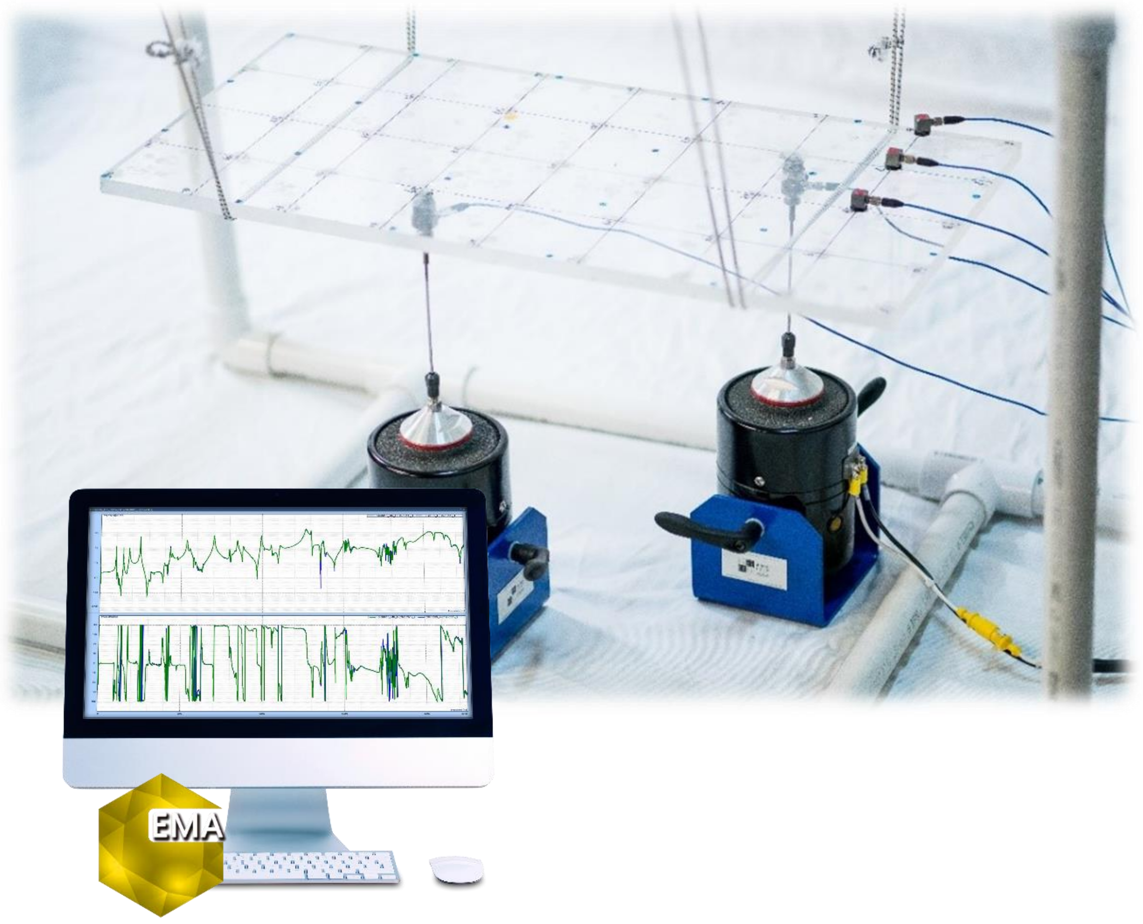
Multi-Resolution Spectrum Analysis

Introduction

Topics Covered



1. Abstract
2. Problem – FFT's with Evenly Spaced Resolution
3. Solution – Multi-Resolution Spectrum
4. Example – Modal Analysis using MR
5. Example – Random Vibration Control Test using MR
6. Possible Future Expansion – TWR, TTH, Shock

Abstract

Narrow band Fast Fourier Transform (FFT) spectrum analysis is widely used to analyze time domain data. Narrow band spectrum analysis by default produces a spectral resolution that is evenly spaced across the bandwidth measured.

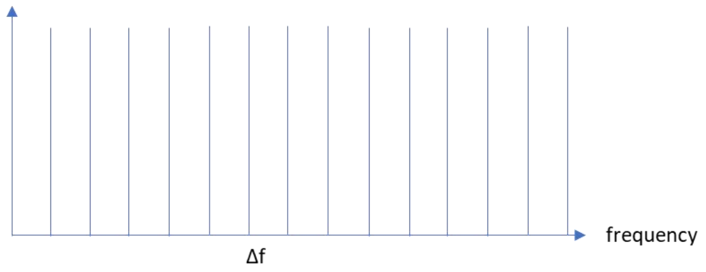
This type of evenly spaced resolution is not ideal in all cases. In many applications it is desirable to have increased resolution at lower frequencies. Crystal Instruments will introduce the concept of multi-resolution spectrum analysis, a software analysis tool that provides improved resolution at lower frequencies along with illustrated use case examples.

Spectrum Analysis

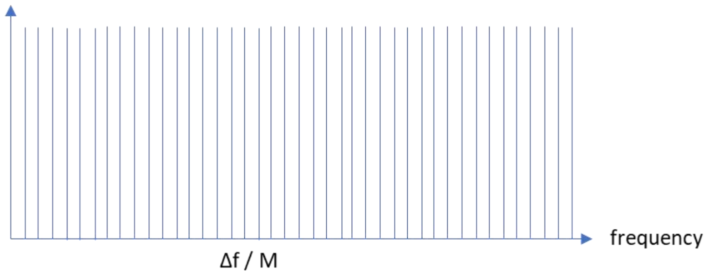
Currently…

- **Cooley–Tukey** FFT (Fast Fourier Transform)
- Resolution is evenly spaced across the entire bandwidth
- Must increase Block Size or decrease Sampling Frequency to improve frequency resolution
- Increasing Block size requires more computation resources and time. In vibration control, the response time of control loop will get much longer when block size is increased

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$



$$\Delta f = \frac{f_s}{Block\ Size}$$



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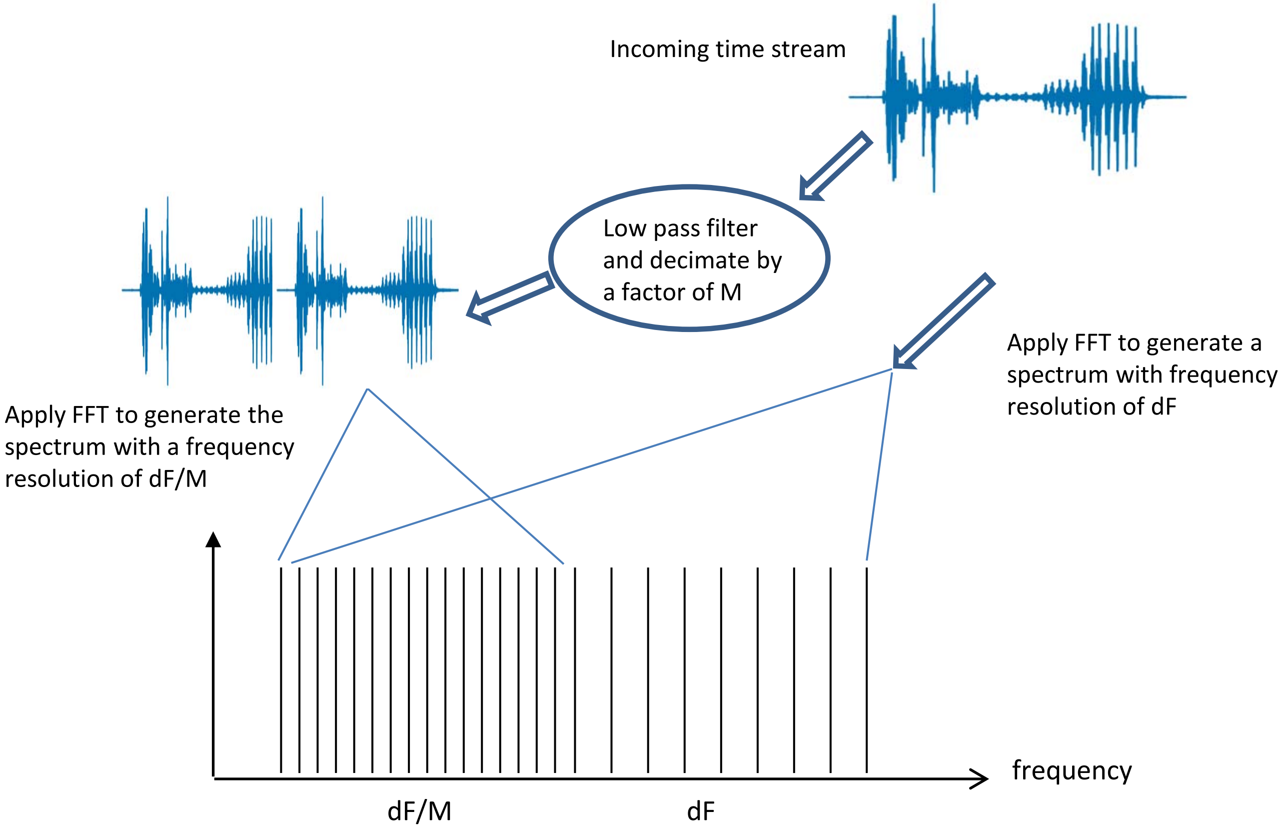
Multi-Resolution Spectrum Analysis

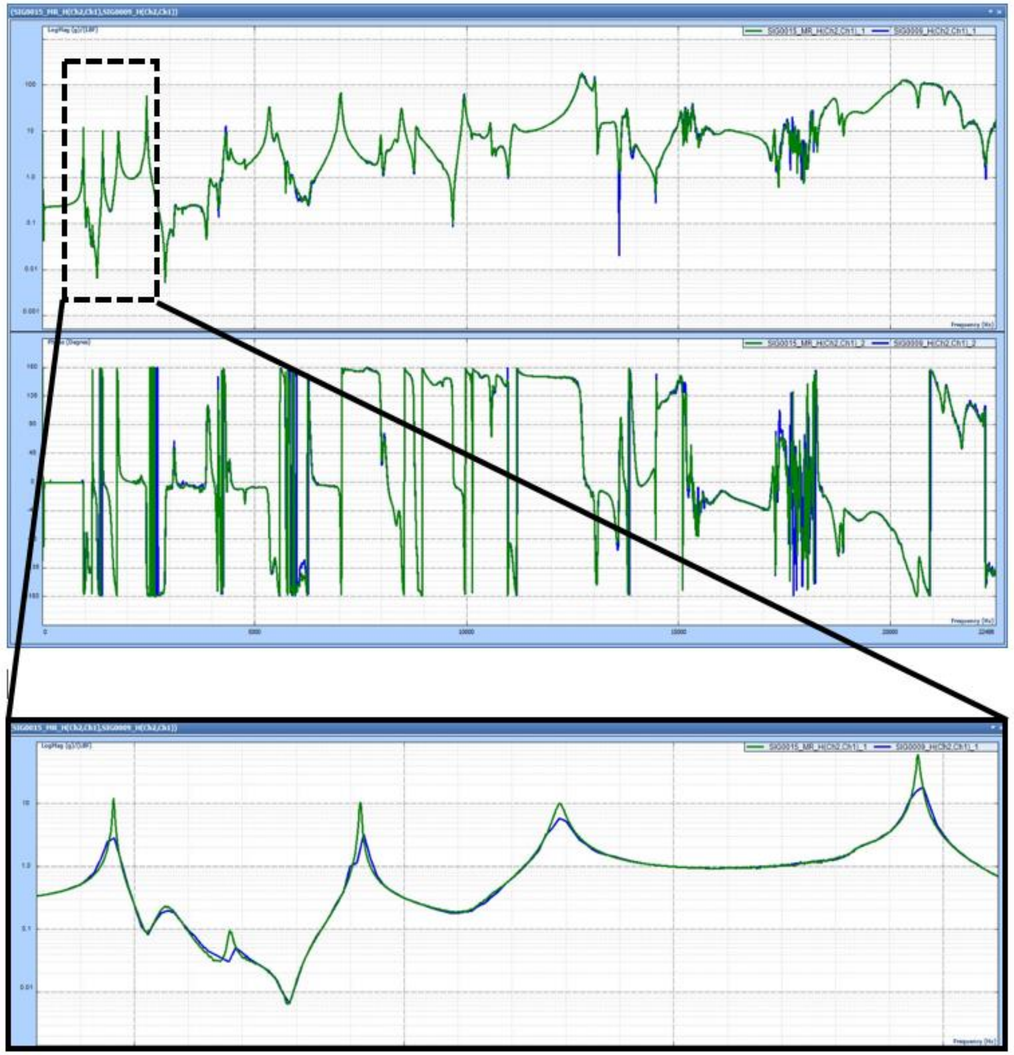
How it works:

- Sample incoming time stream into frames or blocks
- Apply FFT to the time block
- Also apply low pass decimation filter on low-frequency portion and compute the FFT
- Combine both FFT’s to create new spectrum



Example Application: Modal Analysis

- **Test Structure**: Steel plate
- **Boundary Condition**: Free-Free (Supported Vertically)
- **Excitation**: White Noise using a modal shaker (**Hann window** was used to reduce leakage from excitation and response)
- **Sampling Rate**: 51.2 kHz (Bandwidth = 23 kHz)
- **Block size & Spectral Lines**: 4096 / 1800 lines (**Δf = 12.5 Hz**)
- With **Multi-Resolution** enabled: **Δf = 1.56 Hz (up to 2.81 kHz)**
- The comparison plot shows that the **FRF with MR** (**Green Curve**) has much **sharper peaks** as compared to the **FRF with Regular FFT** (**Blue Curve**)



Example Application: Modal Analysis (continued)

Results Compared

Terribly lower than true values

Resonant Frequency	Q estimation using MR	Q estimation using regular FFT	Percentage Different
960.94 Hz	311.069	40.138	87%
1418.75 Hz	313.292	120.452	62%
1789.06 Hz	97.435	52.326	46%
2453.13 Hz	461.059	89.479	81%
5350 Hz	126.317	126.19	0%
8462.5 Hz	172.296	172.73	0%
12725 Hz	94.498	94.065	0%

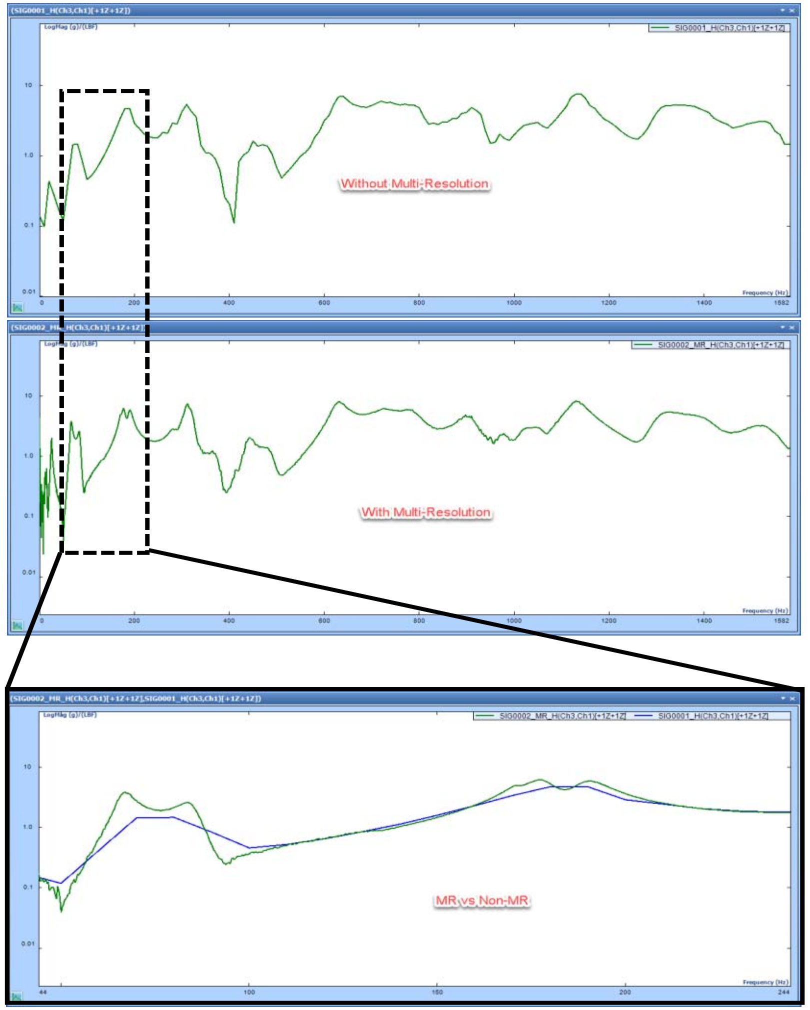
Terribly lower than true values

Resonant Frequency	FRF Amplitude Estimation using MR (g/LBF)	FRF Amplitude Estimation using regular FFT (g/LBF)	Percentage Different
960.94 Hz	12.269	2.832	77%
1418.75 Hz	10.687	3.274	69%
1789.06 Hz	9.993	5.823	42%
2453.13 Hz	60.277	18.42	69%
5350 Hz	33.72	33.74	0%
8462.5 Hz	32.08	32.07	0%
12725 Hz	186.23	186.72	0%

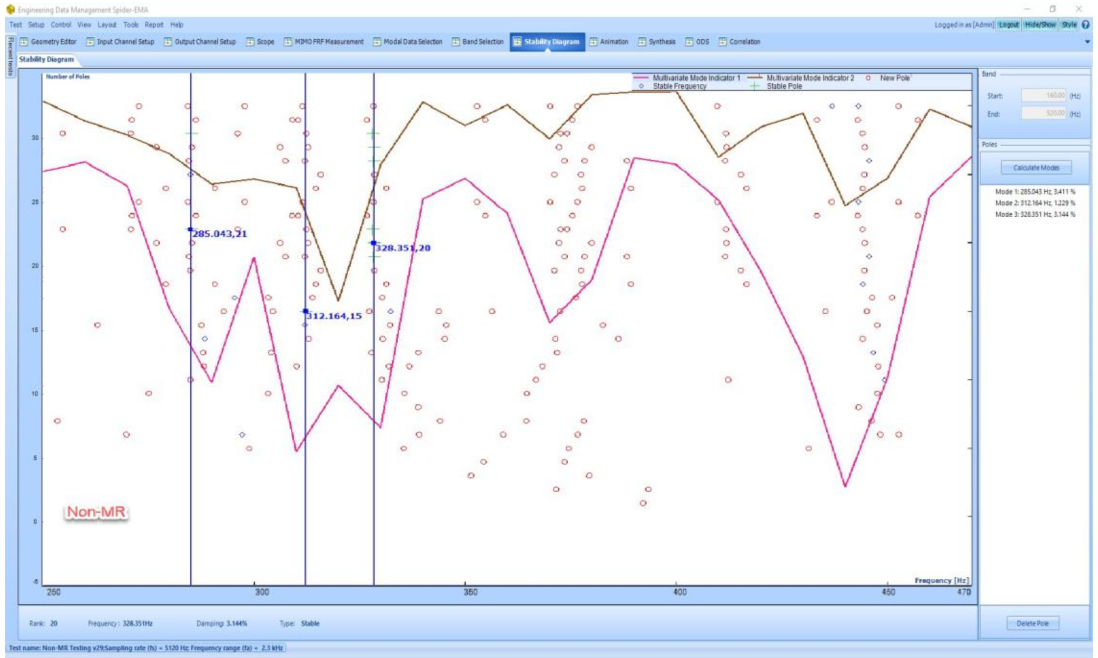
- The tables show that the first four resonance frequencies that are present within the low-frequency cut-off region have a much **higher Q** and **peak amplitude** with the implemented **multi-resolution spectrum**

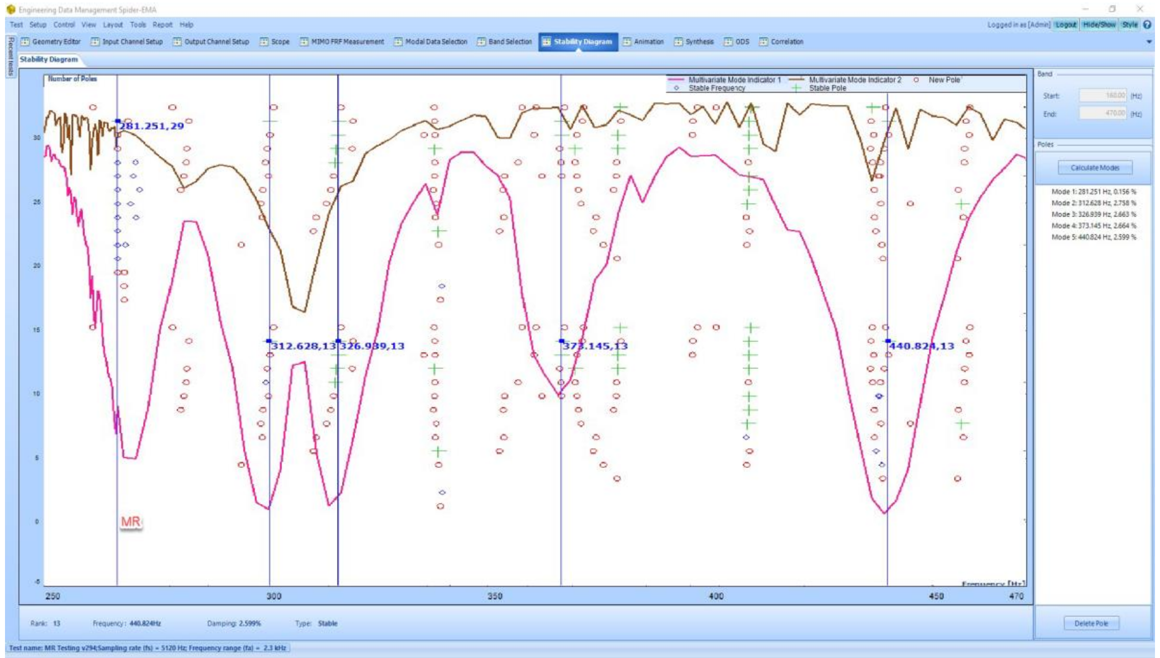
Example Application: Modal Analysis (continued)

- **Test Structure**: Plexiglass plate
- **Boundary Condition**: Free-Free (Supported Horizontally)
- **Excitation**: White Noise using 2 modal shakers (**Hann window** was used to reduce leakage from excitation and response)
- **Sampling Rate**: 5.1 kHz (Bandwidth = 2.3 kHz)
- **Block size & Spectral Lines** : 2048 / 900 lines (**Δf = 2.5 Hz**)
- With **Multi-Resolution** enabled: **Δf = 0.31 Hz (up to 281.25 Hz)**
- The comparison plot shows that the **FRF with MR** (**Green Curve**) shows **4 peaks** whereas the **FRF with Regular FFT** (**Blue Curve**) shows **2 peaks**



Example Application: Modal Analysis (continued)





Stability Diagram of FRF with Regular FFT

Stability Diagram of FRF with Multi-Resolution

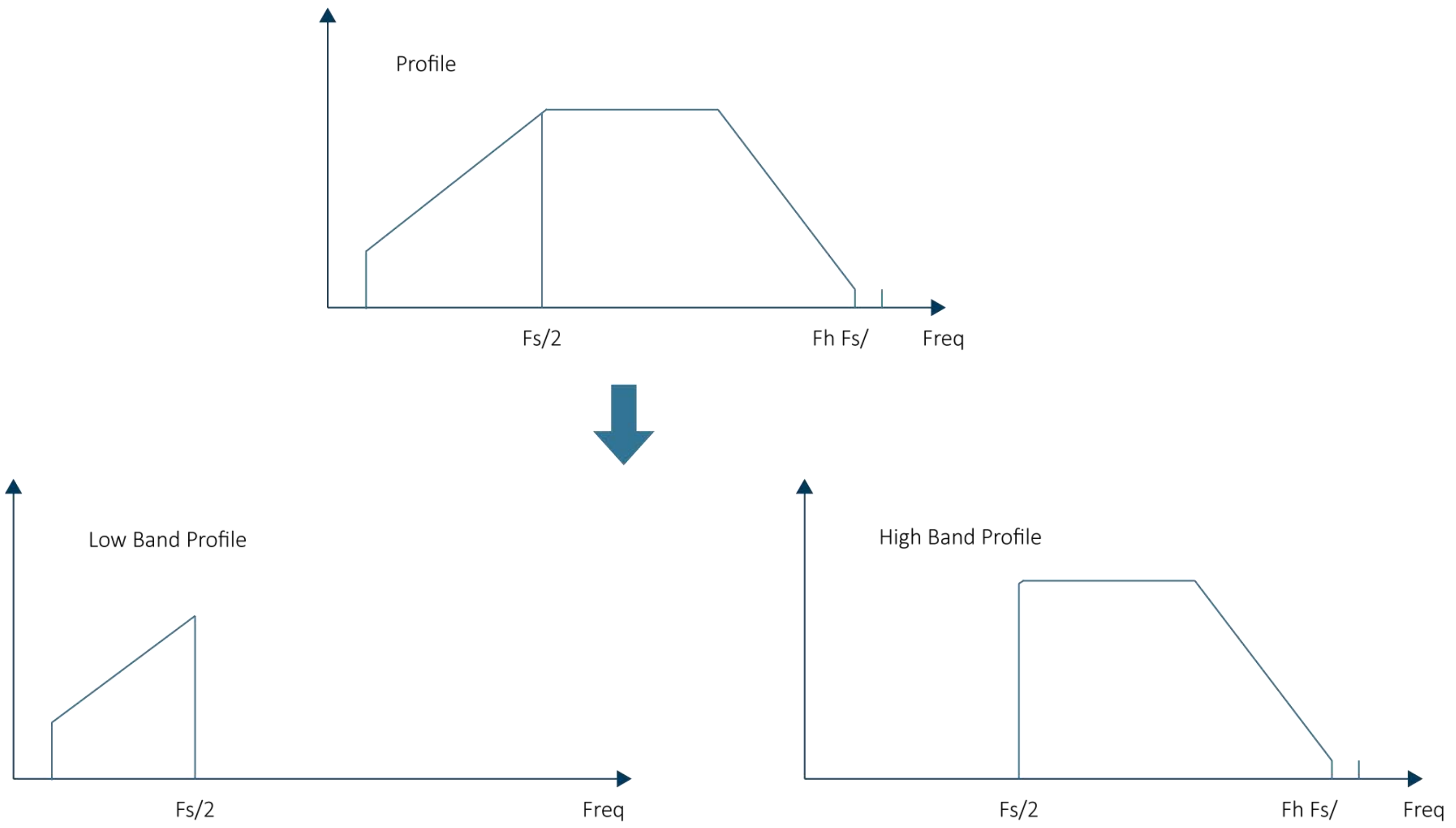
- **The stability diagrams show that the poles (frequency and damping) recognized with Multi-Resolution spectrum are more accurate because of finer frequency resolution**

Example Application: Random Vibration Control

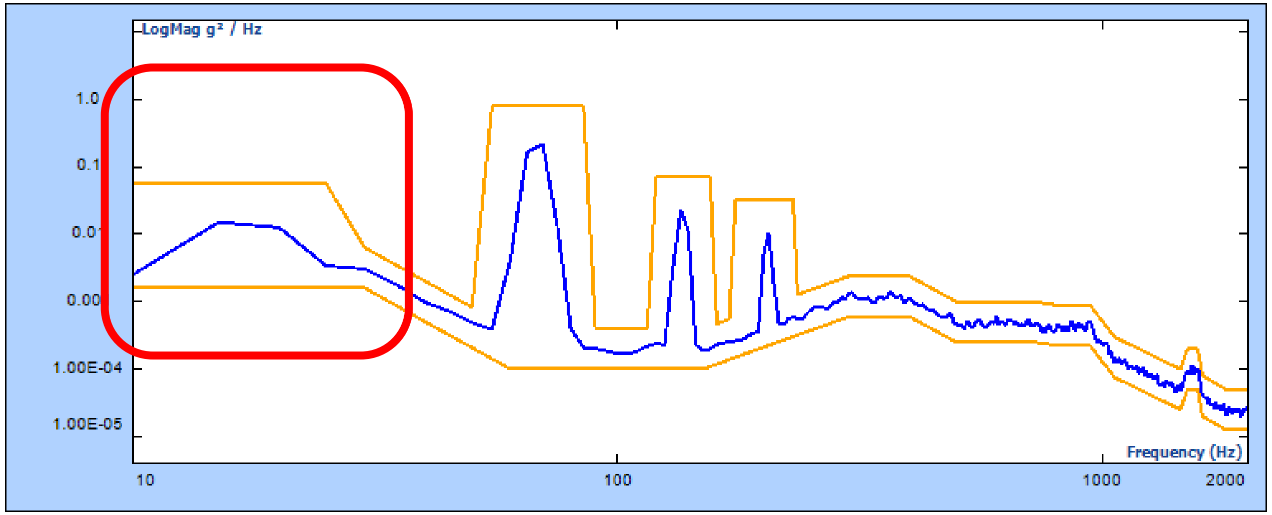
- Multi-resolution spectrum analysis is applied by separating the higher vs. lower frequencies into two parallel control loops

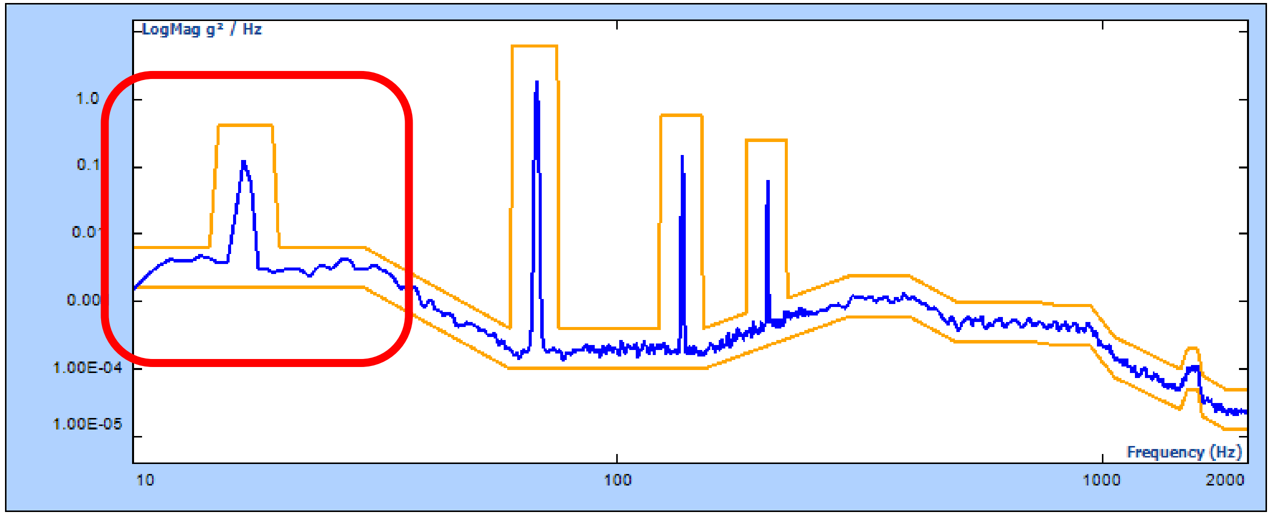
- Improved low band frequency resolution extends to:
  - Drive signal
  - Transfer function matrix

- Competitors without MR must rely on larger block sizes, resulting in:
  - Slow control response times
  - High resource usage (memory and computation)



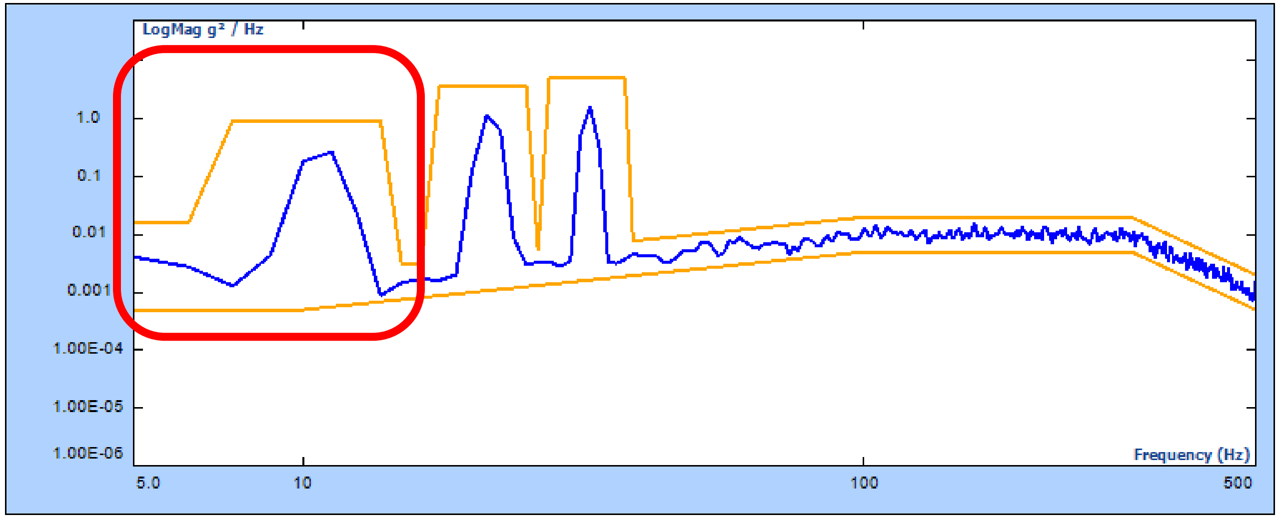
Example Application: MIL-STD-810H (Aircraft, 4-blade C-130)

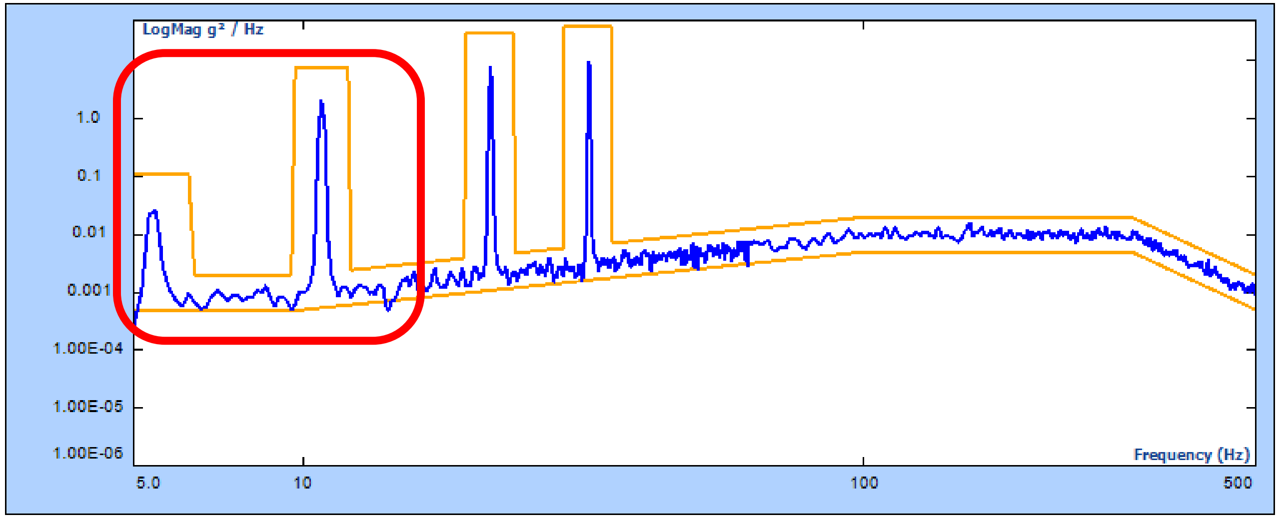




- **Without MR:** hard to see lowest tone (17 Hz)
  - $\Delta f = 5\ \text{Hz}$
- **With MR:** clearly defined sine tones
  - $\Delta f = 5\ \text{Hz} \div 8 = 0.625\ \text{Hz}$
- Block size: 1024 / 400 lines
- Block time: 0.2 s
- Sampling rate: 5.12 kHz
- Sine tones at: 17, 68, 136, 204 Hz

Example Application: MIL-STD-810H (Helicopter, AH-1)





- **Without MR:** hard to separate lowest two tones (5.4 vs. 10.8 Hz)
  - $\Delta f = 1.25 \text{ Hz}$
- **With MR:** clearly separated sine tones
  - $\Delta f = 1.25 \text{ Hz} \div 8 = 0.156 \text{ Hz}$
- Block size: 1024 / 400 lines
- Block time: 0.8 s
- Sampling rate: 1.28 kHz
- Sine tones at: 5.4, 10.8, 21.6, 32.4 Hz

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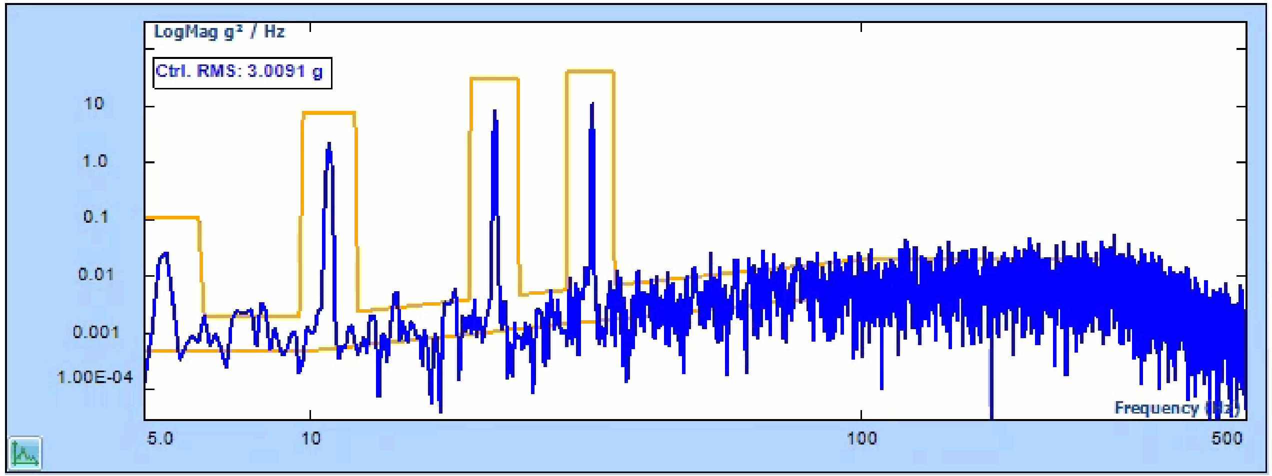
Example Application: MIL-STD-810H (Helicopter, AH-1)

- Competitors without MR must rely on larger block sizes resulting in:
  - Longer block times and pre-test duration
  - Slower responses to safety issues (Abort limits, High RMS changes, etc.)

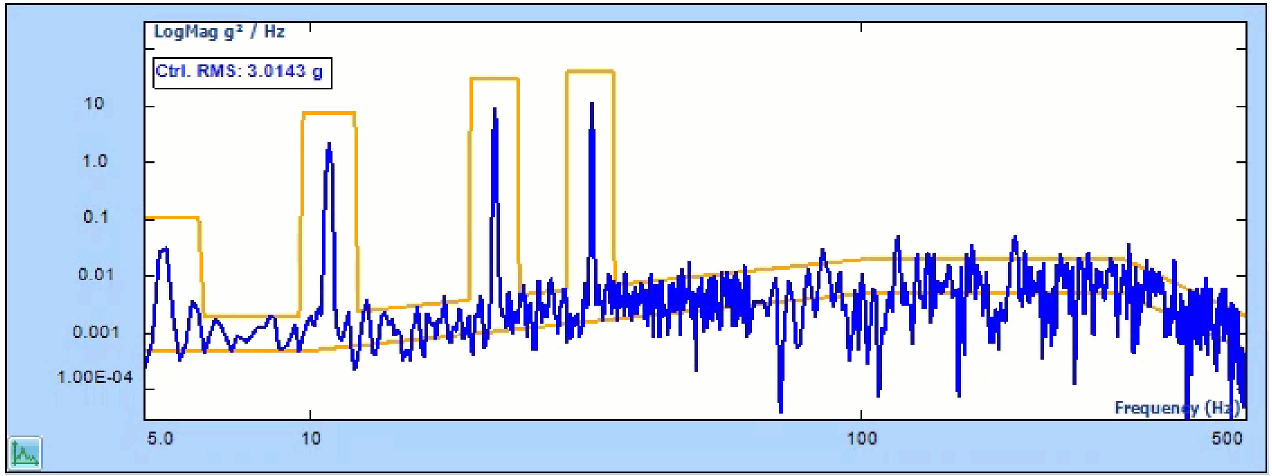
- Matching the previous slide's resolution (0.156 Hz) without MR would mean:
  - 8× longer block size: 8192 / 3200 lines
  - 8× longer block time: **6.4 seconds**

- Responding to aborts / RMS loss of control errors could take **6.4 seconds**!

Without MR: very slow response



With MR, fast response



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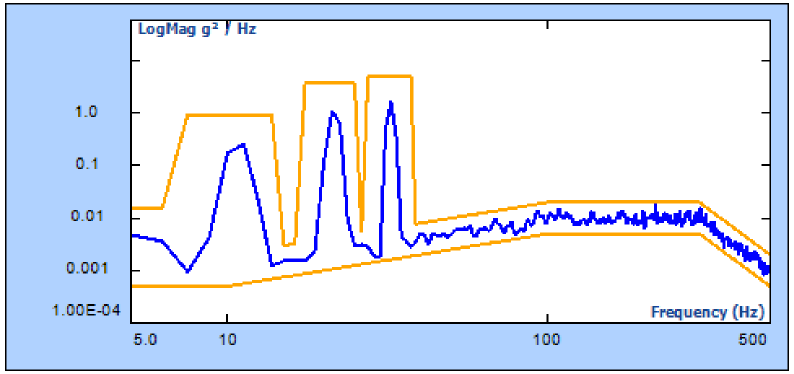
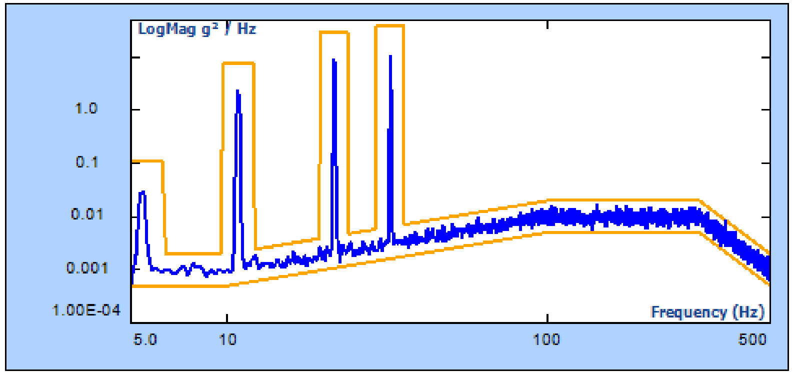
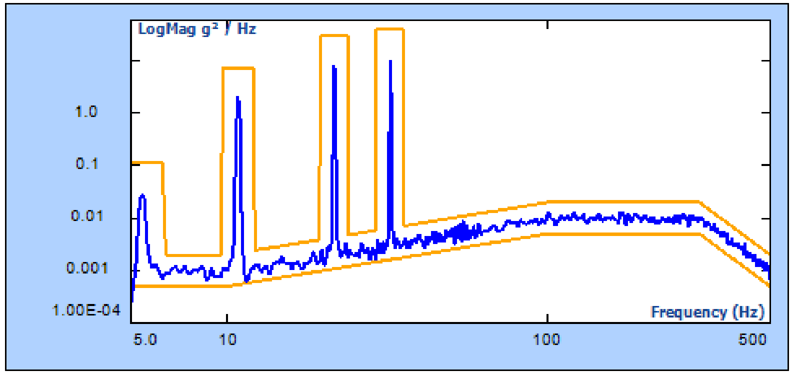
Example Application: MIL-STD-810H (Helicopter, AH-1)







Without MR (low block size)
- Leftmost two peaks cannot be separated
- Poor resolution in low frequencies

Block: 1024 / 400 lines
Block T: 0.8 s

$\boldsymbol{\Delta f}$: 1.25 Hz

$\boldsymbol{f_s}$: 1.28 kHz

Without MR (high block size)
- Slower control response times due to larger block size
- Longer pre-test durations

Block: 8192 / 3200 lines
Block T: 6.4 s

$\boldsymbol{\Delta f}$: 0.15625 Hz

$\boldsymbol{f_s}$: 1.28 kHz

With MR (best of both worlds)
- Good resolution in low frequencies
- Shorter block sizes

Block: 1024 / 400 lines
Block T: 0.8 s

$\boldsymbol{\Delta f}$: 0.15625 Hz / 1.25 Hz

$\boldsymbol{f_s}$: 1.28 kHz

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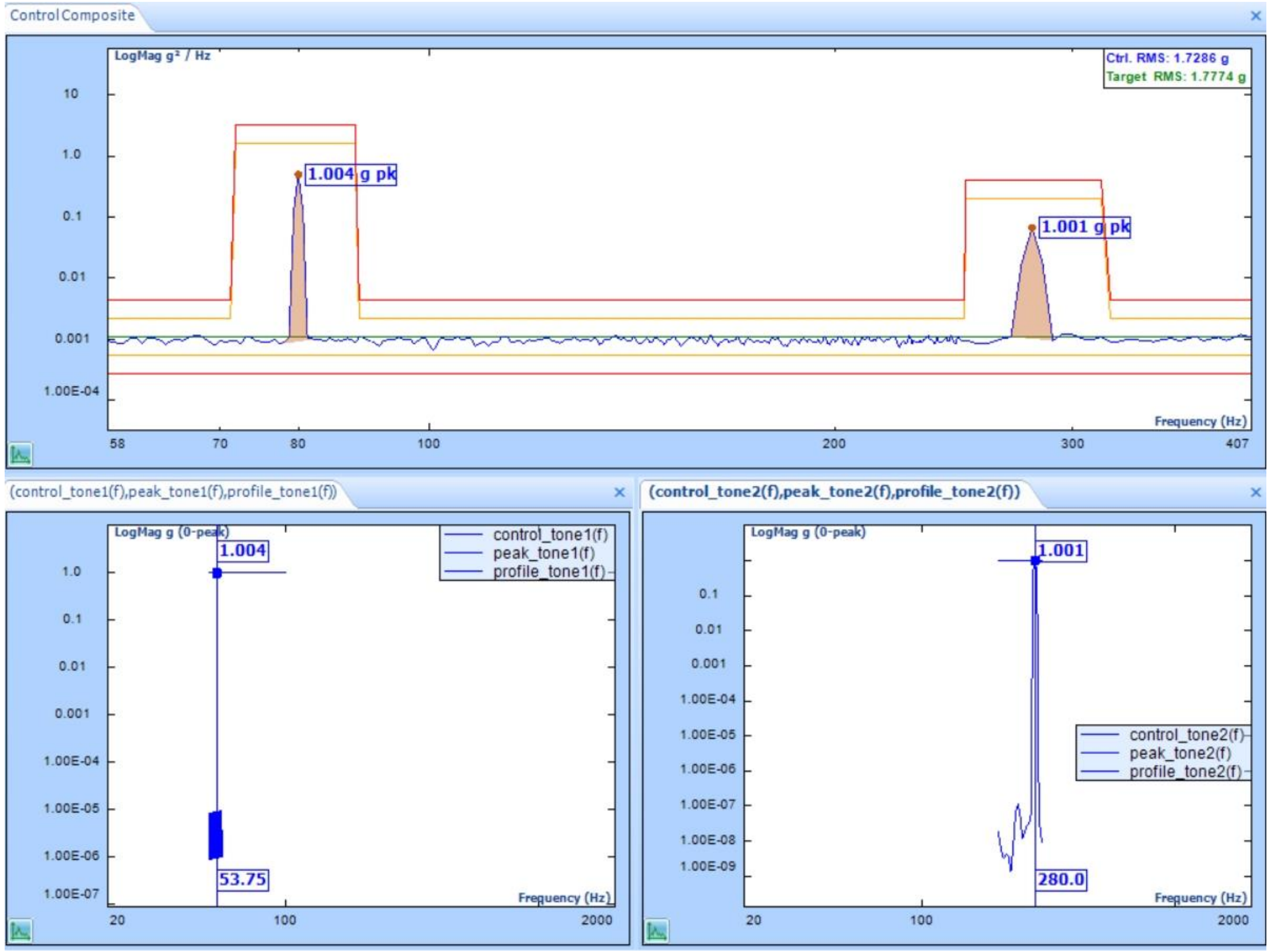
A New Way of Displaying Sine Tone Amplitudes



For two sine tones with the same amplitudes, 1.0 g-pk:

The heights of sine tones on the left side are higher than those on the right because the spectrum resolution is different in these two regions.

The area integrals of these two peaks are the same.

Patent on Multi-Resolution Spectrum Analysis

- Granted Feb. 2019
- Novel
- Wide range of applications
- Possible use in Shock, TTH and TWR control and analysis

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Zhuge et al.

(10) **Pub. No.: US 2019/0041365 A1**
(43) **Pub. Date: Feb. 7, 2019**

(54) **MODAL VIBRATION ANALYSIS SYSTEM**

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(57) **ABSTRACT**

A modal vibration analysis system and corresponding method is provided. A set of one or more exciters is coupled to a structure under test for generating vibrations in the structure. A set of sensors are coupled to the structure at multiple locations for sensing vibrations generated in the structure in response to the excitations. A controller receives sensor signals corresponding to the sensed vibrations from the set of sensors and provides drive signals to the set of exciters such that the sensor signals have a target output spectrum with specified characteristics in multiple designated frequency domains of the spectrum, characterized by a random phase for each frequency. Modal analysis processes digitized sensor signals with a Fast Fourier Transform conducted at two or more specified data sample rates to synthesize a spectrum containing data points with higher resolution for lower frequency range, and regular resolution for higher frequency range. From the multi-resolution spectra, natural frequencies and damping coefficients are determined at each mode, a mode shape at each natural frequency is computed using all measured data from all sensor locations.

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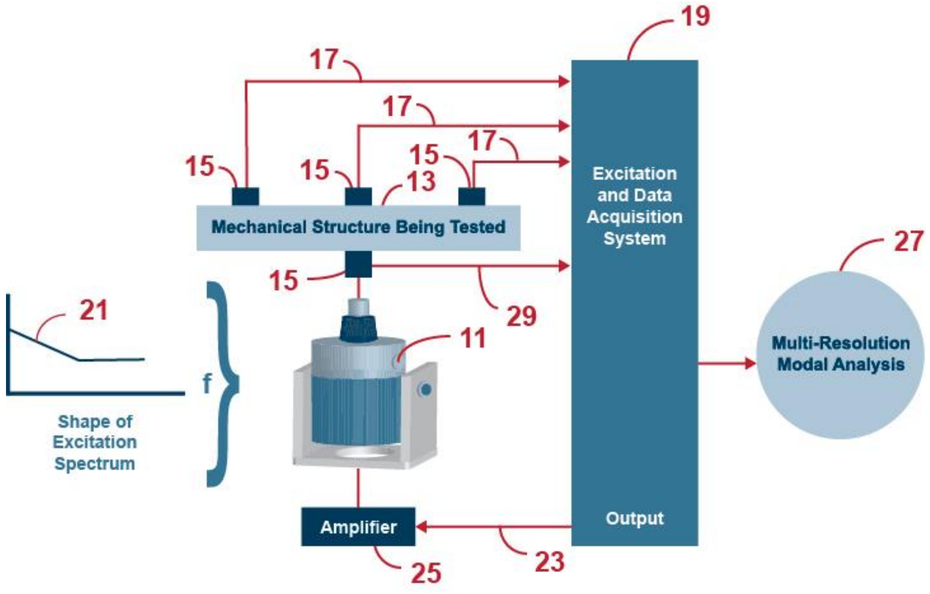
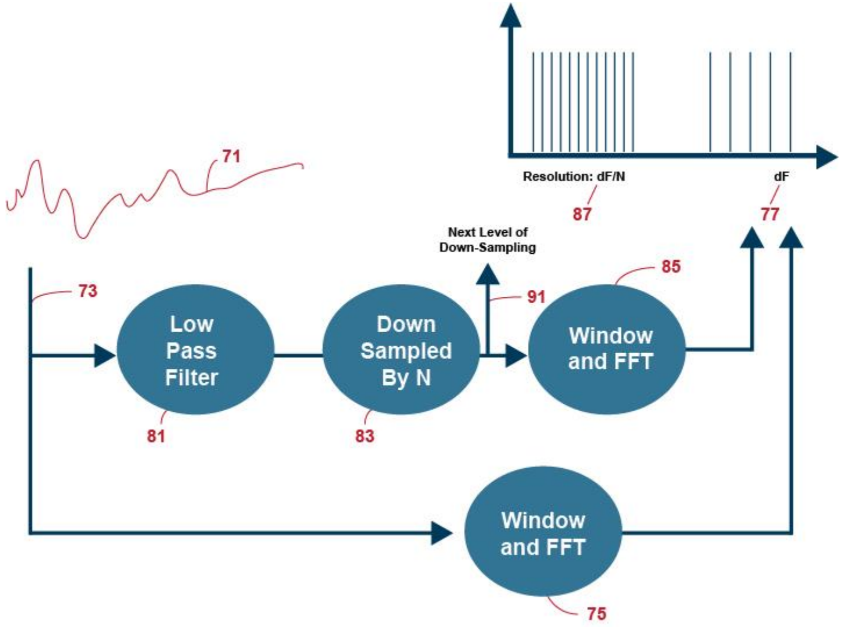
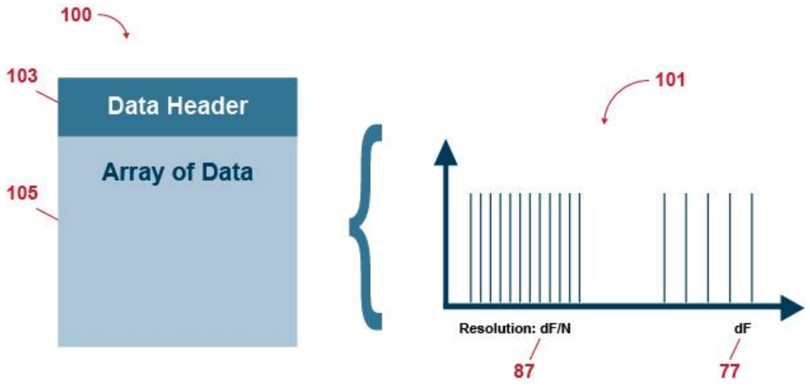
Patent on Multi-Resolution Spectrum Analysis (continued)







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The End

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